**Entropy based OD matrix estimation**

**1. Model**

**1.1 Notation**

Tab. 1 lists general indices, sets, parameters and variables in the problem formulation and solution algorithm.

Tab. 1 indices, sets, parameters and variables.

|  |  |
| --- | --- |
| **Indices** | **Definition** |
|  | Index of origins, |
|  | Index of destinations, |
| **Sets** | **Definition** |
|  | set of origins |
|  | set of destinations |
| **Parameters** | **Definition** |
|  | Generation rate of origin node |
|  | Attraction rate of origin node |
|  | Unit cost of the flow from zone to zone |
|  | The total cost of all the flow in the research region |
|  |  |
|  |  |
| **Variables** | **Definition** |
|  | Traffic flow from zone to zone |

**1.2 Entropy model**

The objective function of the optimization problem is

(1)

which bears three equality constraints of limited inflow, outflow and total cost of transportation. The constraint conditions can be expressed as

(2)

(3)

(4)

Based on equations (1) to (4), a lagrangian function can be constructed as

(5)

where , and refers to the lagrange multipliers. If is large enough, an approximate relation ca be derived from Stirling’s formula, i.e. . Taking the partial derivative of equation (5) with respect to yields

(6)

According to the related knowledge of calculus, the extreme condition is . Thus equation (6) can be converted into an exponential function

(7)

**2. Lagrangian relaxation based solution algorithm**

|  |
| --- |
| **Step 1: Initialization** |
| Initialization iteration number n=0; |
| Initialize the set of Lagrangian multipliers , , and as positive values |
| Step 2: Solve subproblem, equation (5): |
|  |
| Step 3: Update Lagrangian multipliers: |
| 3.1 Calculate the subgradients: |
| Subgradient of generation constraint: |
| Subgradient of attraction constraint: |
| Subgradient of system cost constraint: |
| 3.2 Update Lagrangian multipliers |
| Multipliers of generation constraint: |
| Multipliers of attraction constraint: |
| Multipliers of system cost constraint: |
| Where, is the step size, and |
| Step 4: Termination condition test |
| If is equal to the predetermined maximum iteration number , terminiate the algorithm; otherwise, and go back to Step 2 with updated , , and values |

**3. Numerical example**

The following example is provided to test the effectiveness of the proposed entropy based Origin-Destination matrix estimation model. The observed generation and attraction rates for each zone and the distance between two zones are as shown in Tab. 2.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| O D | 1 | 2 | 3 | Generation |
| 1 | 7 | 17 | 22 | 28 |
| 2 | 17 | 15 | 23 | 51 |
| 3 | 22 | 23 | 7 | 26 |
| Attraction | 28 | 50 | 27 | 105 |

**3.1 GAMS results**



